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BACHELOR OF ENGINEERING IN CIVIL ENGINEERING

SEMESTER TWO

EXAMINATION:

CIVIL ENGINEERING MATHEMATICS 4

YEAR 3

SUMMER 2010

DURATION: 2 HOURS

Date:	Friday 7th May 2010
Time:	14.15pm
Venue:	Main Hall

EXAMINERS: MR BRENDAN MC CANN
MR MARTIN DEEGAN
MR MICHAEL CARR

INSTRUCTIONS TO CANDIDATES

1 FULL MARKS FOR FOUR QUESTIONS
2 ALL QUESTIONS CARRY EQUAL MARKS
3 A TABLE OF LAPLACE TRANSFORMS IS ATTACHED TO THE END OF THIS PAPER

WATERFORD INSTITUTE OF TECHNOLOGY

Question 1

(a) The lateral displacement, X(t), of an object in harmonic motion satisfies the differential equation:

$$\frac{dX}{dt} = 85\cos(20\pi t - \tfrac{\pi}{4})\ \text{mm/sec.}$$

The initial condition X(0) = 15 mm is also satisfied.

(i) Find the particular solution to the differential equation.

(6 Marks)

(ii) Sketch the graph of one complete cycle of X(t).

(6 Marks)

(b) It is known that a body falling in a vacuum is subject to an acceleration of $g = 9.81\ \text{ms}^{-2}$.

(i) Express this as a differential equation in terms of time, t (seconds), and displacement, s (metres). State any assumptions you make.

(4marks)

(ii) Find the general solution to the differential equation.

(4 marks)

(iii) Find the particular solution if the initial velocity is $v_0 = 8.50\ \text{ms}^{-1}$ and the initial displacement is $s_0 = 1.50$ m.

(5 Marks)

Question 2

(a) Use an appropriate integrating factor to find the general solution to the following first-order linear differential equation:

$$\frac{dY}{dt} + 2.50Y = 2.50e^{-4.50t}$$

(7 Marks)

(b) The difference, θ, between the internal temperature of a building and the external air temperature is modelled by the differential equation:

$$\frac{d\theta}{dt} + A\theta = B \quad {}^{0}C/\text{hour},$$

where A and B are non-zero positive constants.

(i) Find the general solution to the differential equation in terms of A and B.

(6 Marks)

(ii) State the numerical condition that A and B must satisfy if the steady-state temperature difference is not to exceed $6.50\ {}^{0}C$.

(3 Marks)

(iii) Sketch the graph of the general solution under the initial condition $\theta(0) = 0\ {}^{0}C$.

(9 Marks)

Question 3

(a) Use the substitution $z = \frac{dy}{dt} - \omega y$ to reduce the second-order differential equation:

$$\frac{d^2y}{dt^2} - 2\omega\frac{dy}{dt} + \omega^2 y = 0$$

to first-order form, where ω is a non-complex real number.

Hence determine the general solution to the differential equation.

(13 Marks)

(b) The variable y satisfies the differential equation

$$\frac{d^2y}{dt^2} + 5.00\frac{dy}{dt} + 6.25y = 0$$

and the initial conditions $y(0) = 1.50$ and $\left.\frac{dy}{dt}\right|_{t=0} = -3.00$.

Find the particular solution for y.

(12 Marks)

Question 4

(a) Confirm that y = sin(2t) is a particular solution to the differential equation:

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 28\sin(2t) + 24\cos(2t).$$

Hence find the general solution to the differential equation.

(8 Marks)

(b) The lateral displacement at the mid-point of a vibrating steel cable satisfies the differential equation

$$\frac{d^2y}{dt^2} + 8.50\frac{dy}{dt} + 86.50y = 24.00\cos(30\pi t) + 15.00\sin(30\pi t).$$

Determine the following quantities:

(i) The undamped natural frequency ϖ_n;

(ii) The damping ratio ξ;

(iii) The damping coefficient α;

(iv) The damped natural frequency ϖ_d;

(v) The forcing function

(5 Marks)

(c) Evaluate the following Laplace Transforms as rational functions in s:

(i) $L(3t^2 - \sin(2t))$;

(6 Marks)

(ii) $L(2e^{-4t}\cos(10\pi t))$.

(6 Marks)

Question 5

(a) Use the method of partial fractions to simplify the following rational function of s:

$$\frac{6s^2 - 3s + 22}{(s-2)(s^2+6)}$$

(4 Marks)

Hence evaluate the inverse Laplace Transform:

$$L^{-1}\left(\frac{6s^2 - 3s + 22}{(s-2)(s^2+6)}\right)$$

(4 Marks)

(b) The lateral displacement, y, of a vibrating structural element is modelled by the differential equation:

$$\frac{d^2y}{dt^2} + 6.0\frac{dy}{dt} + 45.0y = 2.0e^{-3.0t},$$

with the initial conditions y(0) = 0.00 cm and $y'(0) = 3.00$ cm/sec.

(i) Find the Laplace Transform of y.

(9 Marks)

(ii) Use partial fractions and inverse Laplace Transforms to find a formula for y in terms of t.

(8 Marks)

Laplace Transforms of Common functions

$f(t)$	$\mathcal{L}(f)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$

$f(t)$	$\mathcal{L}(f)$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\delta(t)$	1
e^{-at}	$\frac{1}{s+a}$

Properties of Laplace Transforms:

1. $\mathcal{L}\left(\frac{dy}{dt}\right) = s\mathcal{L}(y) - y(0)$

2. $\mathcal{L}\left(\frac{d^2y}{dt^2}\right) = s^2\mathcal{L}(y) - s\,y(0) - y'(0)$

3. First Shift Theorem: (i) If $\mathcal{L}(f(t)) = F(s)$ then $\mathcal{L}(e^{-at}f(t)) = F(s+a)$

   (ii) $\mathcal{L}^{-1}(F(s+a)) = e^{-at}\mathcal{L}^{-1}(f(s))$